PAGE NO :

B. Sc. Sem - I

Mathematics Hons.

Paper — Mat C1

Group - A

Analytical Geometry of Two Dimensions

Book:

Degree Level Analytical Geometry of Two Dimension

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(Bharti Bhawan)

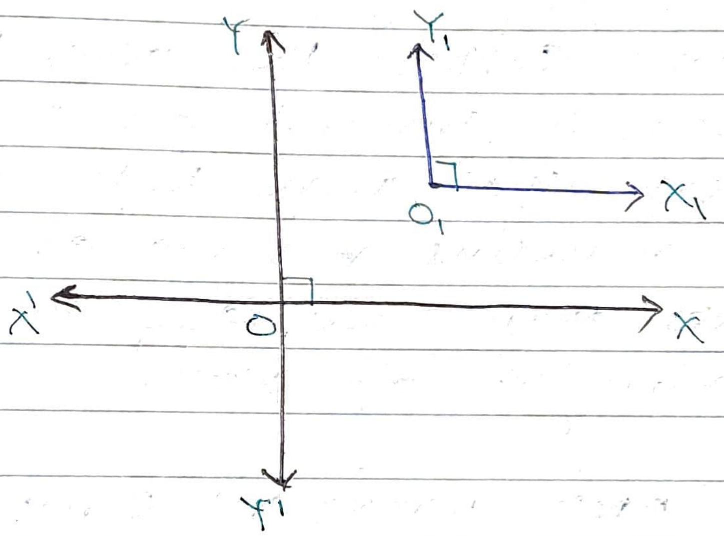
CHANGE OF RECTANGULAR AXES.

FORMS OF TRANSFORMATION

There are three ways of transformation of axes.

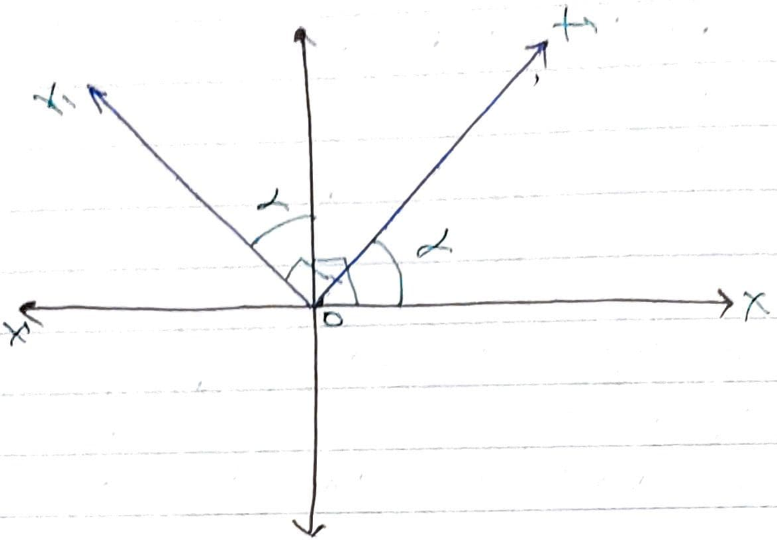
i) Translation of axes or shifting of origin
2) Rotation of axes
3) Translation and rotation both of the axes.

1) TRANSLATION OF AXES OR SHIFTING OF ORIGIN



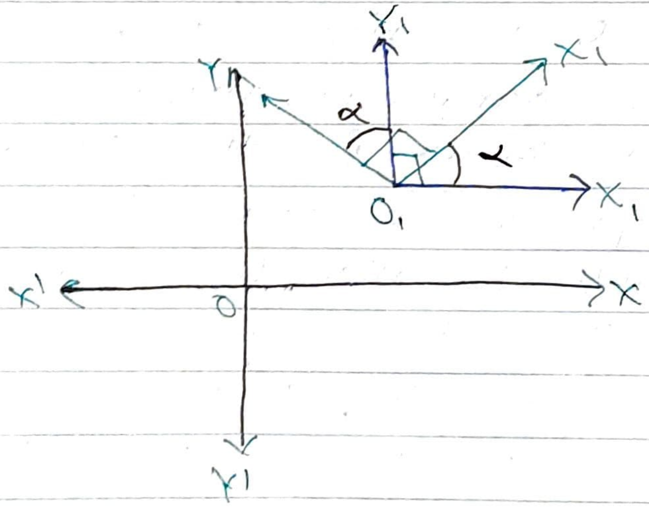
Let us take O as origin and XOX' and YOY' as coordinate axes.
Let O_1 be any point in the same plane through O_1 we draw two straight lines O_1X_1, O_1Y_1 parallel to OX and OY respectively. then the new axes becomes O_1X_1 and O_1Y_1. Such a transformation is called translation of axes or shifting of origin.

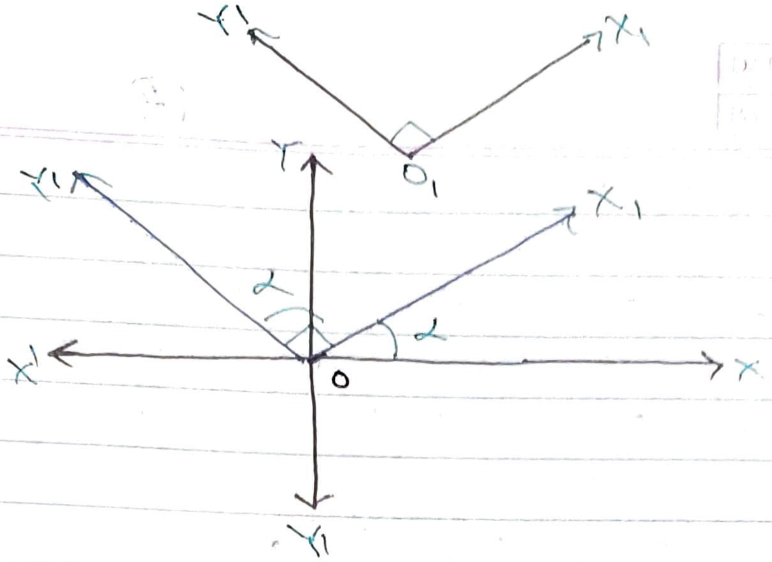
2) ROTATION OF AXES.



Let us take O as origin, OX, OY as coordinate axes. Suppose the axes be rotated about O through an angle α such that the angle between new axes remains ~~x~~ same as the original axes then we say that there is rotation of axes.

3) TRANSLATION AND ROTATION BOTH OF THE AXES.



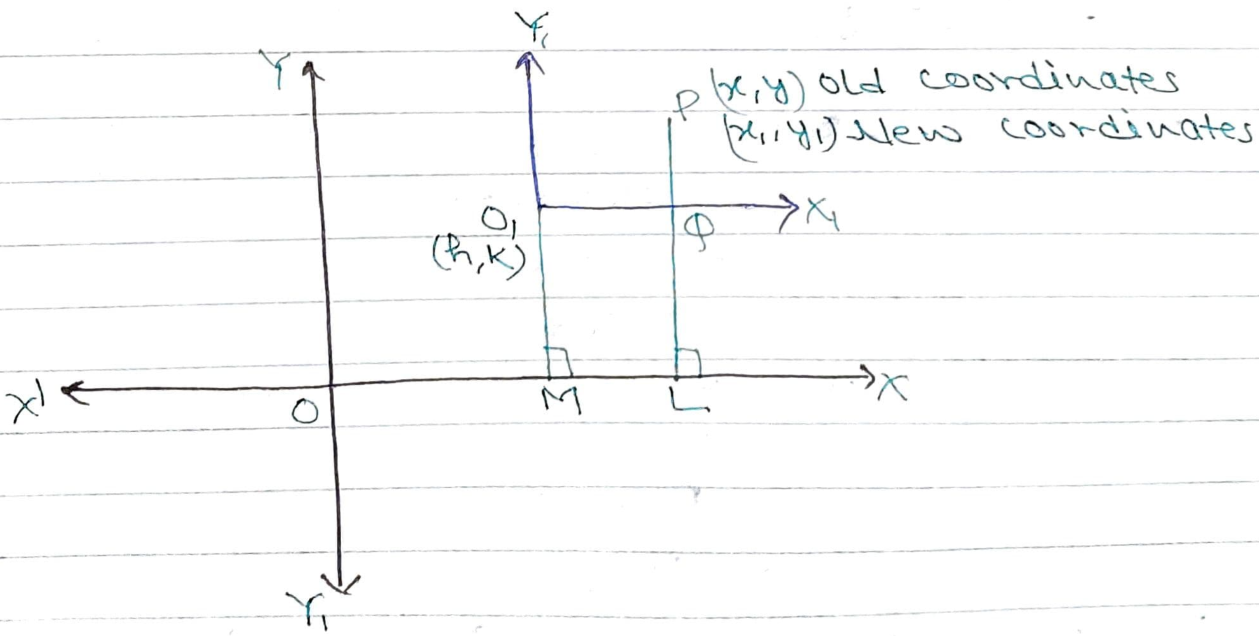


In this case the 2 transformation can be made simultaneously. At first we shift the origin O to the point O_1 and then rotate the axes about O_1 through an angle α.

THEOREM:

To find the coordinates of a point in a plane when the origin is shifted to a new point (h, k) the new axes remaining parallel to the original axes.

Proof:



Let us take O as origin and OX, OY as co-ordinate axes

Let $O_1(h, K)$ be any point in the same plane. Through O_1 we draw the straight lines O_1X_1, O_1Y_1 parallel to the original axes OX, OY respectively.

Let P be any point in the same plane. Let (x, y) and (x_1, y_1) be the coordinates of same pt. P. referred to OX, OY and O_1X_1, O_1Y_1 as co-ordinate axes.

From O_1 we draw $O_1M \perp^r$ upon OX, then

$OM = h$

$O_1M = K$

~~then~~ From P we draw PL $\perp^r$ upon OX which cuts O_1X_1 at Q then

$OL = x$

$PL = y$

and $O_1Q = x_1$

$PQ = y_1$

We have, $x = OL$

$= OM + ML$

$= OM + O_1Q$

$= h + x_1$

$y = PL$

$= PQ + QL$

$= PQ + O_1M$

$= y_1 + K.$

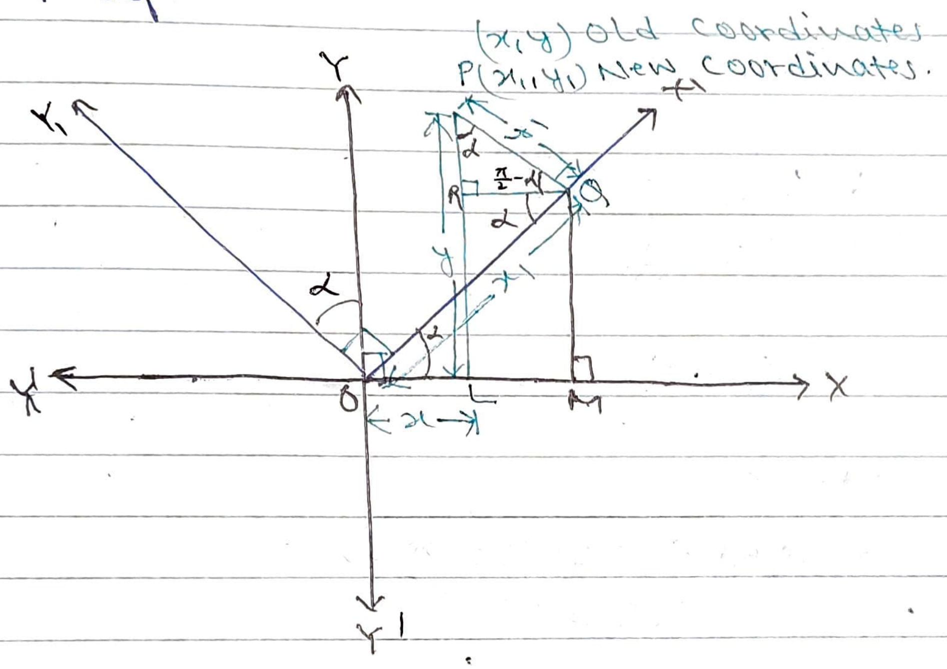
Thus, if origin $O(0,0)$ shifted to a new point $O_1(h, k)$ then we have

$$\boxed{\begin{aligned} x &= x_1 + h \\ y &= y_1 + K. \end{aligned}}$$

THEOREM:

To find the coordinates of a point, when the direction of axes are changed without changing the position of origin and angle between the axes.

Proof:



Let us take O as origin and OX, OY as coordinate axes. Suppose the axes be rotated through an angle α about O In such a way that the angle between new axes will remains same as original axes. Let the new axes becomes OX_1, OY_1

Let P be any point in the same plane From P we draw PL and PQ ⊥r's

upon OX and OX_1 respectively.
Also from Q we draw QR $\perp$r upon PL.
then

$OL = x$, $PL = y$
$OQ = x_1$, $PQ = y_1$

$\because$ QR is $\parallel$rl to OX

$\therefore \angle OQR = \angle QOX$
$= \alpha$

Now, $\angle PQO = \frac{\pi}{2}$.

$\angle PQR + \angle RQO = \frac{\pi}{2}$.
$\angle PQR + \alpha = \frac{\pi}{2}$.

$\therefore \angle PQR = \frac{\pi}{2} - \alpha$.

from rt. ΔPQR.
$\angle PQR + \angle QRP + \angle QPR = \pi$
$\frac{\pi}{2} - \alpha + \frac{\pi}{2} + \angle QPR = \pi$

$\angle QPR = \alpha$.

then from Q, we draw QM $\perp$r upon OX.
From rt. ΔQOM.

$\sin\alpha = \frac{QM}{OQ}$
$= \frac{RL}{x_1}$

$\therefore RL = x_1 \sin\alpha$

$$\cos\alpha = \frac{OM}{OQ}$$

$$= \frac{OM}{x_1}$$

$\therefore$ $OM = x_1 \cos\alpha.$

From rt. ΔPQR

$$\sin\alpha = \frac{QR}{PQ}$$

$$= \frac{LM}{y_1}$$

$\therefore$ $LM = y_1 \sin\alpha.$

$$\cos\alpha = \frac{PR}{PQ}$$

$$= \frac{PR}{y_1}$$

$PR = y_1 \cos\alpha.$

Now,

$x = OL$

$= OM - LM$

$x = x_1 \cos\alpha - y_1 \sin\alpha.$

$y = PL$

$= PR + RL$

$= y_1 \cos\alpha + x_1 \sin\alpha.$

Thus if the axes be rotated through an angle α then we have

$$\boxed{\begin{aligned} x &= x_1 \cos\alpha - y_1 \sin\alpha \\ y &= x_1 \sin\alpha + y_1 \cos\alpha \end{aligned}}$$

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